A3_6 Flying Squirrels: Falling at Terminal Velocity

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Abstract
This paper discusses the physics behind humans landing a fall at terminal velocity. We find that there would have to be a severe increase in human size in order to comfortably survive. A 90 kg human would have to be 2.0 m wide and 8.5 m tall with an area of 17 m² to survive such a fall. We also find that the fluid density would have to be 30 kgm⁻³ to alternatively reduce the terminal velocity of a human enough to survive.

Introduction
It is somewhat known that squirrels cannot die from falling as they can survive the impact from a fall at terminal velocity [1]. Indeed, this is if they land feet first. We aim to extract the physics from this scenario and apply it towards a human falling at terminal velocity. We adjust the necessary parameters to slow a human’s terminal velocity in order to survive.

Method
To begin, we found the terminal velocity equation [2]:

\[ v_t = \sqrt{\frac{2mg}{\rho_f C_d A}} \]  \hspace{1cm} (1)

where \( \rho_f \) is the density of the fluid, which for air is 1.225 kgm⁻³, \( C_d \) is the drag coefficient and \( g \) is the acceleration due to gravity which we took as 9.81 ms⁻². Then we calculated the deceleration during the landing phase of a fall using the simple equation for constant acceleration

\[ a = \frac{v_t - u}{t} \]  \hspace{1cm} (2)

This allowed us to work out the forces involved in a landing by using Newton’s 2nd Law

\[ F = ma. \] \hspace{1cm} (3)

Once the force was calculated we could compare this to the forces that humans can survive. By using the fact gymnasts can survive forces over 12 times their weight during a landing [3], we can substitute this new force back in and find a new, slower terminal velocity that humans can comfortably survive the impact from. This is done by working in reverse, we began with the new force and found the corresponding acceleration using equation (3). Using this new acceleration we found a new terminal velocity using equation (2). Finally, using this new terminal velocity we calculated a new area to slow the terminal velocity using equation (1). Additionally we found a new density of the fluid in a similar way. We held the area constant and used the new velocity as mentioned previously.

Table of values
To carry out the calculations for falling at terminal velocity we needed some values which are in the following table (1).
<table>
<thead>
<tr>
<th>Value</th>
<th>Squirrel</th>
<th>Human</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>0.42</td>
<td>1.70</td>
</tr>
<tr>
<td>Width (m)</td>
<td>0.060</td>
<td>0.41</td>
</tr>
<tr>
<td>Area (m$^2$)</td>
<td>0.024</td>
<td>0.70</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>0.40</td>
<td>90</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.98</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 1: Table of values used in calculations [4], [5], [6], [7], [8].

Substituting these values into equation (1) allows for the terminal velocity of both a squirrel and a human to be found.

Findings

A squirrels terminal velocity was found to be 16 ms$^{-1}$, while a human has a terminal velocity of 59 ms$^{-1}$. If we take $t$ as 0.1 s, a reasonable assumption for a landing given bending of the legs [9], we can say that the deceleration for a squirrel is 160 ms$^{-2}$ and for a human is 590 ms$^{-2}$. This equated to a force on a squirrel equal to 64 N, and 53,000 N on a human. This is significantly higher than the force acting on a gymnast when they land. [3]. By assuming the maximum force a human can comfortably handle as equal to the force acting on a gymnast, we obtain a new maximum force, $F_{new}$, of 11,000 N. Taking this $F_{new}$ we calculated the acceleration using equation (3), taking $a_{new}$ to be 120 ms$^{-2}$, giving a new terminal velocity, $v_{tnew}$, of 12 ms$^{-1}$. From here we can either find the new area by holding the fluid density constant, or we can hold the human area constant and find the fluid density. We calculated the new area, $A_{new}$, to be 17 m$^2$, which is 24 times larger than the assumed dimensions from earlier. We would need a human that is 8.5 m tall while being 2.0 m wide. We also calculated the new fluid density, $\rho_{fnew}$, to be 30 kgm$^{-3}$, which is 25 times the density of air. In these calculations we have assumed that both humans and squirrels are rectangles and that humans can suddenly right themselves and land on their feet.

Conclusion

Humans cannot survive falls through the air at terminal velocity, therefore significant changes in either the anatomical make up of a human, or the density of air would need to be made. A human would need to be 8.5 m tall and 2.0 m wide in order to sufficiently slow the terminal velocity so they can land in 0.1 seconds. Unfortunately, this size human would most certainly not be able to survive, as they wouldn’t have enough muscle density to support their body. For a normal human to be able to comfortably survive the fall the air density would have to be 30 kgm$^{-3}$ which is 25 times larger than normal, which would cause massive changes to life on earth as we know it. More force would be needed for all forms of transportation, and the maximum height humans could breathe at would be lower.

References