P1_8 Escaping the Moons of Mars using a Pogo Stick

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Abstract
The objective of this paper was to investigate the gravitational field strengths of the Martian Moons; Deimos and Phobos’ effect on a pogo stick and it’s rider. It was hypothesised that spring required to escape Deimos would be smaller due to the gravitational field strength being lower. This hypothesis was tested using Hooke’s Law and the Newtonian Laws of gravity to calculate the spring size needed to propel the pogo stick and its rider to a point where it was no longer under the effects of the respective moons gravitational pull. We found this value for the spring decompression to be 1.56 m on Deimos and 5.1 m on Phobos. These values confirmed the hypothesis we made and therefore it would be possible to use a pogo stick to escape the Martian Moons’ gravity.

Introduction
The moons of Mars, Phobos and Deimos, both have exceptionally low surface gravities, $g$, with Deimos having the lower of the two. This results in escape velocity’s of 20 kmh$^{-1}$ for Deimos [2] and 41 kmh$^{-1}$ for Phobos [1], which is approximately one thousandth of the Earths [3]. We resolved to try and find the properties of spring needed to use a Pogo Stick to get off of these Martian Moons by matching their relative escape velocities and determine if this was a viable way to escape the moons. This paper uses Hooke’s Law [6] in conjunction with Newton’s Laws of gravity [7] and the equations of motions [8] to determine this hypothesis.

Theory
The relationship between the properties of the spring, $k$, the spring constant, and $x$, the extension or compression required for the spring to withstand the force, and the force is dictated by Hooke’s Law [6] and is shown in Equation 1.

$$F = -kx$$  \hspace{1cm} (1)

This had to be related to the force due to gravity, Equation 2, and they were set equal to each other and had to be calculated at the radius where the body, pogo stick and its rider, would be unaffected by the gravitational pull of the moon.

$$F = \frac{GMm}{r^2}$$  \hspace{1cm} (2)

Where we assumed a mass, $m$, of 80 kg for the pogo stick and it’s rider, $G$ is the universal gravitational constant [7] and $M$ is the mass of the moon [1] [2]. The radius, $r$, the distance at which the escape velocity is reached, however, needed to be calculated. This was done using Equation 3, where $u$ is the escape velocity and by setting $a$ to be the surface gravity of the given moon, 0.0057 ms$^{-2}$ and 0.003 ms$^{-2}$ for Phobos and Deimos respectively. [1] [2]
\[ v^2 = u^2 + 2ar \quad (3) \]

Once the value of \( kx \) was calculated, the value of \( k \) was taken as 980 Nm\(^{-1} \) [9], as the spring was assumed to be made of steel, and the value of \( x \) was then found to be 1.56 m and 5.1 m for Deimos and Phobos respectively.

Discussion

The spring compression values for Deimos and Phobos show that whilst both being rather large for the size of a pogo stick on Earth would be easier to ride on Deimos than Phobos, as the spring on Phobos has to decompress approximately 3.5 m more than the spring in the pogo stick on Deimos. This extra length required for decompression would create a challenge for the rider when operating the pogo stick as it would require more 'bounces' to provide the compression required. Another issue would be with the production of the size of a spring and this would change the value for \( k \), which is assumed from a value of high grade steel, as the material choice has a factor within the spring constant due to the Young’s Modulus, the measure of the stiffness of a material, being a factor within \( k \) and if incorrect the spring would deform making it useless.

Conclusion

During the testing of the hypothesis that a pogo stick on Deimos and Phobos, the Martian Moons, would be able to produce the force required to escape the given moons gravitational pull, it was found that the size that the spring on the pogo stick would be 1.51 m and 5.1 m for Deimos and Phobos respectively. However, this paper did not consider the following effects and they could be the subject of further analysis:

- the effect that weight reduction would have on the rider cause multiple 'bounces' to create the decompression required to match the escape velocity,
- the composition of the spring for it to withstand the forces it is subjected too during its compression,
- the change in acceleration due to gravity with increasing radius has been ignored.

References