In the children’s picture book ‘Room on the Broom’, the witch is joined by multiple animals on her broomstick during its journey [1]. By assuming the broom is in equilibrium when all the animals are on board, we found that the lift required to keep them in the air would be 850 N ± 30 N. The cauldron could be made of titanium, which would be alloyed with denser materials and have liquid added to reach the required weight.

Introduction
In the children’s book, Dog, Cat, Owl, Frog and the witch carry a cauldron with them, so their vehicle balances in flight. We assume that the witch changes what the cauldron is made of and adds liquid to it to make the broom balance. This increases the lift required to keep the broom in equilibrium. Our aim is to work out what material the cauldron could be made of, the required volume of liquid within the cauldron and the lifting force needed to keep the characters in the air.

Theory and Results
For an object to be in equilibrium, the total clockwise torque must be equal to the total anticlockwise torque:

\[ \sum_{i=1}^{n} m_i \vec{x}_i = 0, \]  

where \( m_i \) is the mass of object \( i \), and \( \vec{x}_i \) is the displacement of that object from the point about which you are taking moments.

The second condition is that the forces must be balanced:

\[ \sum_{i=1}^{n} \vec{F}_i = 0, \]  

where \( \vec{F}_i \) are the forces acting on the broom.

The unknowns in Table 1 were calculated by setting up a pair of simultaneous equations, using Equations 1 and 2. The torque was measured about \( \vec{x}_0 \), and forces were measured with...
downwards being taken as positive. Using this method, the weight of the cauldron and its contents was found to be 80 N ± 10 N, giving a mass of 8.1 kg ± 1.1 kg. The lift force acting perpendicular to the broomstick on the handle’s centre has magnitude of 850 N ± 30 N. Error margins were calculated using the upper and lower boundaries for each value [5].

<table>
<thead>
<tr>
<th>Object</th>
<th>Force (N)</th>
<th>Displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauldron</td>
<td>unknown</td>
<td>0.34 ± 0.005</td>
</tr>
<tr>
<td>Dog</td>
<td>80 ± 5</td>
<td>0.49 ± 0.005</td>
</tr>
<tr>
<td>Bird</td>
<td>0.9 ± 0.05</td>
<td>0.67 ± 0.005</td>
</tr>
<tr>
<td>Cat</td>
<td>80 ± 5</td>
<td>0.69 ± 0.005</td>
</tr>
<tr>
<td>Broom</td>
<td>7.34 ± 0.005</td>
<td>0.75 ± 0.005</td>
</tr>
<tr>
<td>Frog</td>
<td>0.22 ± 0.005</td>
<td>0.89 ± 0.005</td>
</tr>
<tr>
<td>Lift</td>
<td>unknown</td>
<td>0.95 ± 0.005</td>
</tr>
<tr>
<td>Witch</td>
<td>600 ± 10</td>
<td>1.13 ± 0.005</td>
</tr>
</tbody>
</table>

Table 1: The displacements of each of the objects from \( x_0 \) and the downwards force associated with them. Displacements were calculated by taking measurements from figure 1, and the downwards forces were calculated using the masses of the objects. These masses were obtained from various sources [2], [3], [4]. Cat and Dog were assumed to be the same mass, as they look roughly the same size.

The equation relating mass, volume and density is

\[
\rho = \frac{M}{V},
\]

where \( \rho \) is the density of the object, \( M \) is the mass of the object and \( V \) is its volume.

Approximating the cauldron as a hemisphere of inner radius 15 cm and thickness 1 cm, the volume of material required would be 0.00151 m\(^3\). To reach the lowest required mass the material would need a density of 4680 kgm\(^{-3}\). Once the lowest possible required mass is reached, a liquid substance can be added to reach the required weight to achieve equilibrium. Up to 2.2 kg of substance would need to be added into the cauldron to do this.

Titanium (Ti) has a density of 4506 kgm\(^{-3}\). It has a high melting point and is resistant to corrosion, making it more suitable for storing potions than other materials. It can also be alloyed with higher density substances to achieve the required weight [6].

**Conclusion**

When all characters are on board the broom, the cauldron must have a mass of 8.1 kg ± 1.1 kg. The lift force on the broom has a magnitude of 850 N ± 30 N. A possible material to make a cauldron of this mass is titanium. It has a high melting point and is resistant to corrosion, making it suitable for storing potions. It can be alloyed with other materials to achieve the required weight.

**References**


[5] https://uniofleicester-my.sharepoint.com/:x:/g/personal/lah42_student_le_ac_uk/EcB-t6GgDyNDsA9huWrk7moBldklty6IGSX_cWoHFJ5Fg?e=aGpopo [Accessed 24 September 2019]