A1_7 Can a skydiver without a parachute be saved?

Author: Gemma Keen, Laura Evans, Tamsyn Evans, Jennifer Gosling

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

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Abstract
This report investigates the speed of freefall of skydivers to determine whether or not it is possible to save someone who falls from a plane without a parachute. We conclude that it is possible, but that the rescuer must jump from the plane within 47.6 seconds of the person who fell.

Introduction
A skydiver’s motion is governed by the combination of forces due to gravity and air resistance. Air resistance is proportional to a body’s cross-sectional area and velocity squared. The drag force due to air resistance is given by equation 1 [1],

$$ F_D = \frac{1}{2} C_D \rho A v^2, $$

where $A$ is the cross-sectional area in the plane perpendicular to the direction of travel, $v$ is the velocity of the skydiver relative to the air, $\rho$ is the density of air (approximately 1.28 kgm$^{-3}$ [1]) and $C_D$ is the drag coefficient, which we will assume to be 1.2 for skydivers to simplify the calculation, as this $C_D$ is appropriate for a person [6].

In an ideal situation, it is recommended that a parachute be released by 610m from the ground in order to achieve a safe landing and to have time to react to any problems that may occur [2]. Many professional skydives are from 3048m altitude [3]. Therefore, to save a person who falls from a plane without a parachute, the rescuer has only 2438m in which to catch up with them. It is also worth noting that parachutes are often designed to support more than one person, tandem skydives being quite common.

As all objects are accelerated by the same amount due to gravity, the only difference in speed of two objects will be due to air resistance. For a person trying to catch up with someone who has fallen, they must therefore seek to have a lower cross sectional area to reduce the affect of air resistance and fall faster.

Analysis
We will assume that the person leaving the plane first will make a spread eagled position in the air, thus maximising their surface area. The second person we will assume to fall head first, minimising their surface area, as illustrated in fig.1 below.
Fig. 1 [4, 5]

The surface area perpendicular to the airflow of a person in the spread eagle position we have estimated as 1.8m (approximately 6 feet) multiplied by 0.5m (approximately shoulder width) plus 0.0375m² (approximate surface area of the head).

For the person travelling head first, the surface area perpendicular to the airflow was estimated as 0.0375m² (surface area of the head) plus 0.0525m² (approximate visible area of the shoulders).

The mass of both people we have assumed to be 75kg.

To calculate the time taken for someone to fall 2438m, we will use the constant acceleration equations (equations 2 and 3 below) for segments of 0.001 seconds, modifying the acceleration and velocities after each calculation. This was achieved by creating an iterative program in IDL (program available on request).

\[ s = ut + \frac{1}{2}at^2 \]  \hspace{1cm} (2)
\[ v = u + at \]  \hspace{1cm} (3)

where \( s \) is the vertical distance travelled, \( u \) is the initial velocity at the start of the segment, \( a \) is the acceleration downwards and \( v \) is the velocity at the end of the segment.

The acceleration is given by equation 4 (Newton’s second law of motion), which combines the acceleration due to gravity with the deceleration due to air resistance.

\[ a = g - \frac{c_dA\rho v^2}{2m} \]  \hspace{1cm} (4)

where \( g \) is the acceleration due to gravity, and \( m \) is the mass of the body.

A segment of code is included here to illustrate the method used,

\begin{verbatim}
WHILE distance GE 610 DO BEGIN
  time=time+0.001
  acceleration=9.81-{(0.5*area*density*0.5*(velocity^2.))/mass}
  distance=distance-(velocity*0.001+0.5*acceleration*0.001^2.)
  velocity=velocity+acceleration*0.001
  pts=pts+1
ENDWHILE
\end{verbatim}

Using these calculations, it was found that the person falling without a parachute would take 78.5 seconds to reach the minimum height to safely deploy a parachute. The person falling while minimising their surface area would take only 30.9 seconds to reach that altitude.

Conclusion

Therefore, a skydiver with a working parachute can indeed catch up with someone who falls from a plane and save them, if they jump from the plane within 47.6 seconds. This is ample time in which to react to such an emergency, assuming that the height of the jump is 3038m and the parachute deployment height must be 610m. A single parachute may support two people, so the falling person can indeed be saved if a skydiver with a working parachute catches them in time.

However, in these calculations we have neglected to include effects that would complicate things, such as the fact that both skydivers will also travel a potentially significant distance horizontally. To extend this study, those effects would need to be taken into account.

References
[1] Physics for scientists and engineers, Tipler
[5] http://lh6.ggpht.com/_9Kz_FA2hG_U/Smm7vQ3fwuI/AAAAAAAAATw/MuDMPpC7Si4/Skydivers%20shutterstock_32696518_thumb%5B1%5D.jpg