Journal of Physics Special Topics

A2_1 Deep Space 1: Staying Cool

M. Bryan, J. Forster and A. Stone

Department of Physics and Astronomy, University of Leicester. Leicester, LE1 7RH.

Oct 19, 2012.

Abstract
In order to undertake an interstellar mission, as with normal space missions, cooling is of great importance as there are no ways for convection or conduction to remove excess heat. This paper will study the ability of radiative cooling to remove heat from a system and show that it is unsuitable for dealing with the energy requirements of such a vessel and indeed cooling systems cannot even handle the transportation of said heat.

Introduction
Interstellar travel as modelled under current technology requires relativistic speeds to be considered plausible. However while the problem of energetics is often brought up to reach such speeds should it ever become possible to produce sufficient energy one emergent problem would be that of cooling the waste heat produced by such a generator. In such a journey removing heat through ejecting materials seems less plausible as as much material must be conserved as possible so only radiative emission of heat can be considered.

Theory
This system, while in space, has to rely on radiation as a method of removing heat. This can be calculated by using the Stefan-Boltzmann law

$$P_r = e\sigma A(T^4 - {T_c}^4), \tag{1}$$

where P_r is the radiated power, e is the emissivity, A is the radiating area, σ is Stefan's constant, T is the temperature of the radiator and T_c is the temperature in space. By assuming that the ship will be a cylinder of length $l = 50$m and radius $r = 10$m which is powered by a Pressurised Water Reactor (PWR) of efficiency $\eta_R = 0.32$ [1] and propelled by a number of High Power Electric Propulsion (HiPEP) ion thrusters [2] at maximum output of $F_I = 670$mN at $P_I = 39.3$kW with an efficiency $\eta_I = 0.75$. These are chosen for their low fuel requirements which could be vital on any long journey. To make the journey as fast as possible the vessel should be maintaining acceleration either to speed up or slow down the ship at all times. A constant acceleration of 1g, where g is the gravity on Earth, is chosen and the craft is presumed to have a similar density to a space shuttle with maximum load [3] of $\rho_s = 6.67 kg/m^3$. This gives the following equations for the power required to propel the ship and the excess heat produced.

$$P_e = (\pi r^2 l \rho_s g)\frac{P_I}{F_I} \tag{2}$$

$$P_T = \frac{P_e(1-\eta_R)}{\eta_R} + (1-\eta_I)P_e \tag{3}$$

which assumes that any other sources of heat are negligible compared to the outputs of the reactor and thrusters.

Radiating Heat
The radiators on the vessel must be capable of emitting a value of $P_R = P_T$ to prevent a build-up of heat. By making the radiators out of aluminium anodised with gold, which has a high emissivity $e = 0.82$ [4], the minimum total area that is required to emit the heat produced by the craft can be calculated.The temperature of the radiator, T, will also be important in determining the power radiated which will be assumed to be the same temperature as coolant departing the PWR at $T = 317$K. These values are put into a final equation which takes the form

$$A = \frac{\frac{P_e(1-\eta_R)}{\eta_R} + (P_e(1-\eta_I)}{e\sigma(T^4 - {T_c}^4)} \tag{4}$$

which is taken from equations 2 and 3. Solving equation 4 by inputting the variables produces a final size for the radiative area is given as $3.05 \times 10^8 m^2$ which is a size vastly larger than the ship's surface area along its length which is calculated as $3141 m^2$. If proportionality is maintained as the ship gets larger the problem gets worse as the surface area increases proportionally to r^2 while the power output increases with r^3.

Heat Transfer

To top off the problems in the example given the heat has to be transferred from all reactors to the radiators in order to remove it. The most likely candidate for a coolant is water, given that a PWR is the source of heat, which has a $C_P = 4180$J which increases in temperature from 291K to 317K creating a temperature change of 26K. This means the mass flow rate is given by

$$\Delta m = \frac{P_T}{\Delta T C_P} \tag{5}$$

which gives a result of 1.32×10^6 kg/s which is vastly heavier than the ships weight of 1.05×10^5kg in a single second.

Discussion

As shown by the results the modelled cooling method is incapable of handling the heat produced by relativistic vessels. This holds true as even if the radiators are perfect black bodies and the generators operate at their carnot efficiency the discrepancy between cooling and heating is too large a gulf to bridge. So it is not just enough to develop a new method of power generation but a suitable cooling method must also be developed to ever make such a journey possible.

References

[1] https://netfiles.uiuc.edu/mragheb/www/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Pressurized%20Water%20Reactors.pdf accessed on 18/10/2012.

[2] J.E.Foster,T.Haag, M.Patterson, G.J.Williams, J.S.Sovey, C.Carpenter,H.Kamhawi, S.Malone and F.Elliot "The High Power Electric Propulsion (HiPEP) Ion Thruster" NASA/TM2004-213194 AIAA20043812.

[3] D.R. Jenkins(2006). "Space Shuttle: The History of the National Space Transportation System"Voyageur Press. ISBN 0-9633974-5-1.

[4] http://snap.fnal.gov/crshield/crs-mech/emissivity-eoi.html accessed on 18/10/2012