A1_10 We’re dead, Jim!

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Abstract
This report looks into the plausibility of the U.S.S. Enterprise escaping from the gravity of a black hole in the film Star Trek (2009). This report estimates the mass of the black hole in this instance and the distance the Enterprise is from it. It concludes that the energy required to free the Enterprise is about 2.23 x 10^{27} J, requiring 24 x 10^6 tonnes of matter to turn to energy. This is approximately a thousand times larger than the total mass of the Enterprise, and hence the ship will be unable to escape in this manner.

Introduction
In the 2009 film Star Trek there is a scene where the Enterprise is caught in the gravity of a black hole. The solution they devise is to eject the warp core (the main power source) and detonate it behind the Enterprise so that the force of the explosion allows the ship to escape [1].

This report will discuss the possibility of the Enterprise escaping in this way. In the film, ships can time travel by passing through black holes, which also have a massive gravitational pull. Time travel via black holes is a convention used in many sci-fi, and as there are no other apparent distinctions between these and real black holes, The black hole in Star Trek is modelled as having the same properties as a real black hole.

Discussion
To determine the possibility of escape the gravitational potential energy of the Enterprise has to be determined. The first stage is to work out the mass $M$ of the black hole [2]:

$$M = \frac{r_s c^2}{2G},$$

(1)

where $G$ is the gravitational constant, $r_s$ is radius from the centre of the black hole to the boundary where light can escape (the Schwarzschild radius, or radius of the black hole), and $c$ is the speed of light. $r_s$ can be calculated as, in the film, it forms around a ship called the Narada with a diameter approximately the same as that of the ship [1]. The Narada's radius is not readily available, however fan websites have estimated it to be about 6 miles long [3]. Using a scale diagram [4], the radius of the ship (and thus the black hole) was estimated to be $r_s \approx 1.9$ km. From this, Equation 1 gives $M = 1.28 \times 10^{30}$ kg.

The next step is to determine the distance between the black hole and the escaping Enterprise. In order to do this, it has been assumed that the Enterprise which appears in the 2009 film is the same ship as the ship in the original TV series (with updated graphics) for which there are technical specifications available [5].

![Figure 1. An example frame from Star Trek showing the Enterprise trying to escape from a black hole (taken at 1:46:52) [1].](image)
width of the Enterprise as seen by the observer. This is made clear in Figure 2.

![Diagram](image)

Figure 2. Diagram showing the geometry of the system perceived in Figure 1. O represents the observer, and \( r_e \) and \( r_b \) are the distances from the observer to the Enterprise and the black hole respectively.

The width of the enterprise is \( w_e = 127.1 \) m [5], and so using the diagram it can be said that

\[
\tan \frac{\theta}{2} = \frac{w_e}{2r_e} = \frac{r_s}{r_b} \tag{2}
\]

The ratio \( r_e/r_b = w_e/2r_s \) can be satisfied by many different values of \( r_e \) and \( r_b \), rendering a calculation of \( r_b - r_e \) impossible from this data alone. Figure 2 can also be used to describe a similar situation that can be seen in Figure 1: that of the distance between the sensor array (blue disk in Figure 1) and the bow of the ship. This distance \( d \equiv 102 \) m, and the width of the sensor array \( w_s \equiv 17.8 \) m, using a scale diagram of the Enterprise [6]. If it is assumed that the angle the components subtend in the image are proportional to their size in the image then by analogy with Figure 2 it can be said

\[
\frac{w_{ea}}{w_{sa}} = \frac{w_e}{w_s} \frac{r_e + d}{r_s} \tag{3}
\]

where \( w_{ea}/w_{sa} = 6.53 \) is the ratio of the widths of the Enterprise and signal array as measured from Figure 1. Equation 3 yields \( r_e = 728 \) m, and from Equation 2 \( r_b = 21.8 \) km.

The gravitational potential energy of the Enterprise at this point is [7]

\[
U = \frac{GMm_e}{(r_b - r_e)} = 7.71 \times 10^{21} \text{ J}, \tag{4}
\]

where \( m_e = 1.9 \times 10^6 \text{ kg} \) is the mass of the Enterprise [5]. After the warp core is jettisoned, it is somewhere between the Enterprise and the black hole, and is so a maximum distance of \( D \equiv 21 \) km away when it detonates. This detonation needs to release an energy of

\[
E = U \frac{4\pi D^2}{A}, \tag{5}
\]

for the ship to escape, where \( A = 137.5 \times 139 = 19 \times 10^3 \text{ m}^2 \) is a very roughly assumed surface area of the stern of the Enterprise as seen by the explosion. Equation 5 gives \( E = 2.23 \times 10^{27} \text{ J} \), requiring a mass of \( 24 \times 10^6 \text{ tonnes} \), using \( E = mc^2 \).

**Conclusion**

This report has determined that the Enterprise would not have escaped the black hole because the exploding warp core would need to have had a thousand times more mass than the entire Enterprise in order to produce the energy required to push the ship to safety. There were many sources of error in this report, such as the measurements made on Figure 1 and in the values taken from the scale diagram of the Enterprise. This however would only introduce a small error, not big enough to account for the very large difference between the Enterprise’s mass and the fuel needed for escape. Also, the Enterprise depicted in the 2009 film differs aesthetically from the ship in the original Star Trek TV series (although it is technically meant to be the same ship). The assumption that the length scales have remained the same over the years may introduce another small error, which again would not greatly affect the final result.

**References**